Downdraft Table Particle Capture Efficiency Calculation

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To calculate capture efficiency of a downdraft table, the presented method used a particle tracking model with some assumptions on the flow field.

Assumptions:

1). Flow around the table: uniform, but not necessarily vertical, horizontal velocity components of the airflow can be considered. No pressure gradient in the flow field.

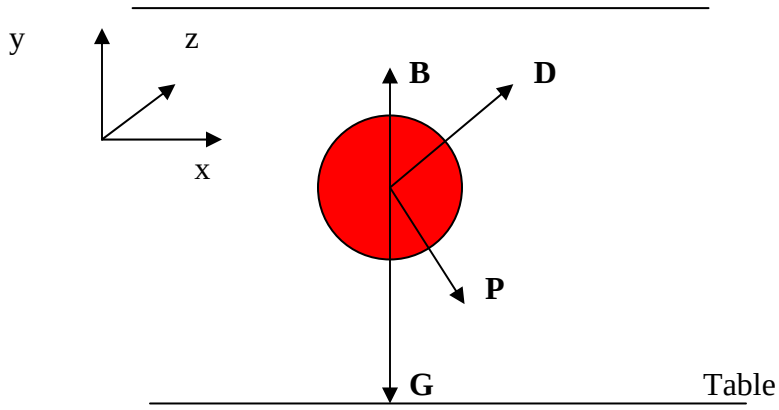
2). Particles can have non-zero initial velocity.

3). Particles will not collide to each other

4). When a particle hit table, the particle is considered captured.

Particle Movement Calculation

Forces that exerted on a moving particle:



G: Gravity force
B: Buoyancy force
D: Drag Force
P: Pressure Force (caused by the pressure gradient in the flow field).

Neglected forces are: Additional inertia force, Basset Force, Magnus Force, Saffman Forces, Lift force.

Particle Motion Equation:

$$\sum \mathbf{F} = m\mathbf{a} = \mathbf{G} + \mathbf{B} + \mathbf{D} + \mathbf{P}$$

$$m\mathbf{a} = \frac{1}{6}\pi\, d^3 \rho_p \frac{d\mathbf{u}_p}{dt}$$

$$\mathbf{G} = \rho_p \mathbf{g} V$$

$$\mathbf{B} = -\rho_g \mathbf{g} V$$

$$\mathbf{D} = \frac{1}{8}\pi C_D d^2 \rho_g \left|\mathbf{u}_g - \mathbf{u}_p\right| (\mathbf{u}_g - \mathbf{u}_p)$$

$$\mathbf{P} = 0$$

C_D can be calculated by either Clift and Gauvin's formula or Putnam's formula:
Clift and Gauvin's formula

$$C_D = f\frac{24}{\mathrm{Re}_r}$$

$$f = 1 + 0.15\,\mathrm{Re}_r^{0.687} + 0.0175\frac{\mathrm{Re}_r}{1 + 42500\,\mathrm{Re}_r^{-1.16}}$$

Putnum's formula

$$C_D = f\frac{24}{\mathrm{Re}_r}$$

$$f = 1 + \frac{1}{6}\mathrm{Re}_r^{2/3}$$

Where Re_r is the Reynolds number based on relative velocity

$$\mathrm{Re}_r = \frac{\rho_g\left|\mathbf{u}_g - \mathbf{u}_p\right|d}{\mu}$$

When expressed in three directions form, the formula becomes

X: $$\frac{du_{px}}{dt} = \frac{3C_D\rho_g}{4d\rho_p}\sqrt{(u_{gx}-u_{px})^2 + (u_{gy}-u_{py})^2 + (u_{gz}-u_{pz})^2}\,(u_{gx}-u_{px})$$

Y: $$\frac{du_{py}}{dt} = -\frac{\rho_p - \rho_g}{\rho_p}g + \frac{3C_D\rho_g}{4d\rho_p}\sqrt{(u_{gx}-u_{px})^2 + (u_{gy}-u_{py})^2 + (u_{gz}-u_{pz})^2}\,(u_{gy}-u_{py})$$

Z: $$\frac{du_{pz}}{dt} = \frac{3C_D\rho_g}{4d\rho_p}\sqrt{(u_{gx}-u_{px})^2 + (u_{gy}-u_{py})^2 + (u_{gz}-u_{pz})^2}\,(u_{gz}-u_{pz})$$

Numerical Solution

Simple explicit first order differencing scheme:

Use $\frac{d\mathbf{u}_p}{dt} = \frac{\mathbf{u}_p^{n+1} - \mathbf{u}_p^n}{\Delta t}$ and get:

$$u_{px}^{n+1} = \Delta t\left\{u_{px}^n + \frac{3C_D\rho_g}{4d\rho_p}\sqrt{(u_{gx}-u_{px}^n)^2+(u_{gy}-u_{py}^n)^2+(u_{gz}-u_{pz}^n)^2}\,(u_{gx}-u_{px}^n)\right\}$$

$$u_{py}^{n+1} = \Delta t\left\{u_{py}^n - \frac{\rho_p-\rho_g}{\rho_p}g + \frac{3C_D\rho_g}{4d\rho_p}\sqrt{(u_{gx}-u_{px}^n)^2+(u_{gy}-u_{py}^n)^2+(u_{gz}-u_{pz}^n)^2}\,(u_{gy}-u_{py}^n)\right\}$$

$$u_{pz}^{n+1} = \Delta t\left\{u_{pz}^n + \frac{3C_D\rho_g}{4d\rho_p}\sqrt{(u_{gx}-u_{px}^n)^2+(u_{gy}-u_{py}^n)^2+(u_{gz}-u_{pz}^n)^2}\,(u_{gz}-u_{pz}^n)\right\}$$

This method requires relatively small time steps to keep the solution stable, currently 0.0001s is found sufficient.

Capture Efficiency Calculation

Capture efficiency(percentage) can be easily obtained by calculate a single particle's drift distance in horizontal directions. For example, if a particle is released from X_0, Y_0, Z_0 in space, we just need to calculate how much distance the particle has drifted from X_0, Z_0, given the fixed vertical falling height. Assume the trajectory of the particle cross (X0-X_d,0, Z_0-Z_d), then the capture percentage is simply

$$C = \frac{L - X_d}{L}\frac{W - Z_d}{W} \times 100\%$$

where L and W are length and width of the draft table.

In the spreadsheet, the ranges of particle releasing and landing are represented by two rectangles.

Nomenclature

G: Gravity force vector
B: Buoyancy force vector
D: Drag Force vector
P: Pressure Force (caused by the pressure gradient in the flow field) vector
a: Accelaration of particle vector
m: Mass of particle
V: Volume of particle
d: Particle diameter
ρ_p: Density of particle
ρ_g: Density of Gas (air)
C_D: Drag coefficient of particle
f: Resistance factor
Re_r: Reynolds number based on relative speed
$\mathbf{u}_g$: Gas velocity vector (u_{gx},u_{gy},u_{gz} are velocity components in x,y,z directons)
$\mathbf{u}_p$: Particle velocity vector
$\mathbf{u}_p^{n+1}$: Particle velocity vector in the new time step
Δt: Time step
L: Length of the downdraft table (center area where the flow is drawing downwards)
W: Width of the downdraft table (center area where the flow is drawing downwards)
X_0, Y_0, Z_0: Release location of particle
X_d,Z_d: Drift distance in X, Z directions.
C: Capture efficiency

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User Input

Table Dimension		
Length X	24	in
Width Z	24	in

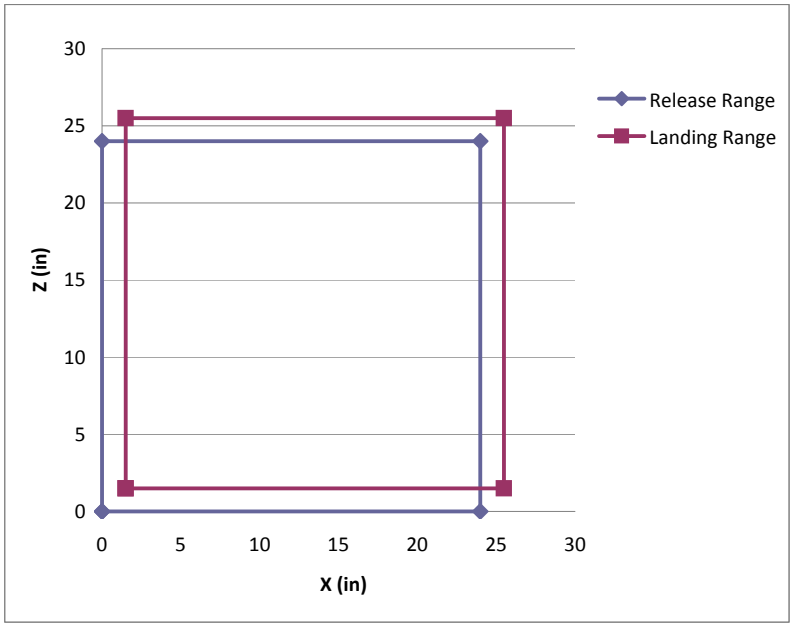
Airflow Condition around Table		
Air Density	1.1614	kg/m^3
Air Viscosity	1.84E-05	Ns/m^2
Vertical Speed Vy	**-50**	ft/min
Horizontal Speed Vx	15	ft/min
Horizontal Speed Vz	15	ft/min

Particle Information		
Size	5	micron
Density	1000	kg/m^3
Release Height	**5**	in
Particle Initial Velocity		
Vertical Speed Vy	0	ft/min
Horizontal Speed Vx	0	ft/min
Horizontal Speed Vz	0	ft/min

Output

Drift Length X	1.50	in
Drift Length Z	1.50	in
Drift Length	2.12	in
Capture Percentage	**87.92**	%

	Release Range		Landing Range	
	X	Z	X	Z
Corner 1	0	0	1.50	1.50
Corner 2	24	0	25.50	1.50
Corner 3	24	24	25.50	25.50
Corner 4	0	24	1.50	25.50
Corner 1	0	0	1.50	1.50



Particle Trajectory

User Input

Table Dimension		
Length X	24	in
Width Z	24	in

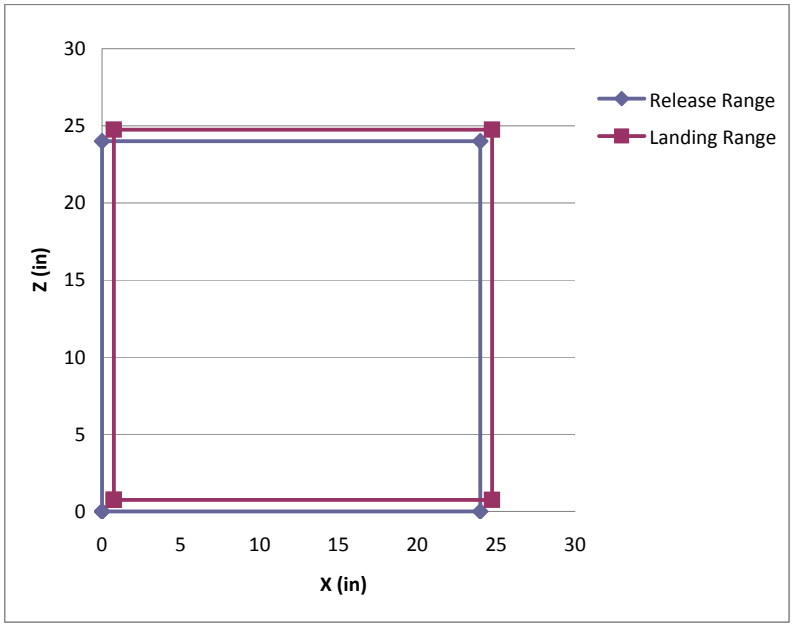
Airflow Condition around Table		
Air Density	1.1614	kg/m^3
Air Viscosity	1.84E-05	Ns/m^2
Vertical Speed Vy	**-100**	ft/min
Horizontal Speed Vx	15	ft/min
Horizontal Speed Vz	15	ft/min

Particle Information		
Size	5	micron
Density	1000	kg/m^3
Release Height	**5**	in
Particle Initial Velocity		
Vertical Speed Vy	0	ft/min
Horizontal Speed Vx	0	ft/min
Horizontal Speed Vz	0	ft/min

Output

Drift Length X	0.75	in
Drift Length Z	0.75	in
Drift Length	1.06	in
Capture Percentage	**93.85**	%

	Release Range		Landing Range	
	X	Z	X	Z
Corner 1	0	0	0.75	0.75
Corner 2	24	0	24.75	0.75
Corner 3	24	24	24.75	24.75
Corner 4	0	24	0.75	24.75
Corner 1	0	0	0.75	0.75



Particle Trajectory